NOTATION AS THE CHIEF BARRIER AND THE FAILURE TO FOCUS ON THE PSYCHOLGY OF LOGIC EDUCATION (PLE)

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At the present time we do not have any academic communities that are taking a full-fledged programmatic interest in what I will call the "psychology of logic education." The main thrust of my presentation will call attention to four key challenges that need to be addressed and fully resolved, so that this new area, now known as the Psychology of Logic Education (PLE), can come into existence.

The first major challenge comes from the need to introduce a *repeat* of what happened in the history of numbers. This challenge points to the evolution of notation. Historically, the significant major systems used to write numerals looked to the efforts made by the Babylonians, the Chinese, the Maya, and the Arabic translators who relayed the fundamental insights coming from India. In effect, there would be no Psychology of Mathematics Education (PME), as we know it today, if we still used Roman numerals. Instead, today what is done is grounded in fundamental structure. Fortunately, the rock bottom essentials that have been built into the Hindu-Arabic numerals that we are now using are such that this structure now inhabits an abacus that has been carried to the mental level, a *mental abacus* that is anchored to the real number line.

Likewise, the first major step that leads to the existence of PLE depends on making a major shift in the notations that are now being used for the Propositional Calculus, otherwise called the Logic of Atomic Sentences (and, or, if). These notations are not grounded in the deep interrelations that inhabit the fundamental structure. Instead and in contrast, the rock bottom essentials that we need are found in the Box-X notation devised by Peirce in 1902, a notation that today stands under a shadow so large and so dense that it is commonly ignored. Note that the X-stem Logic Alphabet (XLA) that I devised is a shape value notation that consists of 16 letter shapes. X-stem is now seen as a direct continuation of Peirce's Box-X. Like Arabic numerals, X-stem is grounded in fundamental structure. In effect, as required by the needs of the Propositional Calculus, X-stem has been coded so that every one of the 16 letter shapes is given meaning by always *mentally framing it against an all-common standard square,* itself serving as the root shape that can be subjected to interrelations-sensitive symmetry operations.

The second major challenge comes from the need to introduce a *replay* of what has become standard curriculum when the use of numbers is taught from K to 12. This is basic for PME as we know it. The sequence of lessons in the curriculum parallels the main line order of the cognitive stages in Piaget. Sensorimotor games include finger counting. Physical objects are also counted, such as beads, trees, whatever. Dienes Blocks model the decimal system. Diagrams depict assortments of numerals in many ways. And so forth, as the students continue to cover the increasing complexity in the number operations introduced across the curriculum.

Likewise, the second major step that leads to PLE depends on constructing a curriculum that will parallel the cognitive stages in Piaget. This is precisely what becomes possible, given the way X-stem has been designed. Start very early and include sensorimotor finger positions. Then go to marble boards, followed by dot cards. Now introduce the shapes of the 16 letter shapes, but at this stage make no reference to the code that gives logical meanings to the letter shapes. Continue with the hand-held symmetry models, namely the logic bug, the flipstick, the clock compass, and then the logical garnet. Note that the far end of what is happening here, far beyond the scope of this presentation, is that the complexity in the use of these symmetry models can be continued all the way to the college level, even to graduate school.

The third major challenge comes from a need that is nicely noted in what happened more than a thousand years ago, when Roman numerals still carried the day, but at a time when Arabic numerals were slowly being carried into Europe. Someone named Gerbert went to Spain, learned how to write Arabic numerals, returned to France, but as long as he lived, he never learned how to add and multiply with them. I call this the "Gerbert Barrier." Here I am calling attention to a basic assumption that PME takes for granted. We expect our students to do both --- not only to write the Hindu-Arabic numerals but also to be able to operate on them, namely, to activate the four standard operations in arithmetic.

Likewise, the third major step calls on the same assumption just mentioned for PME. PLE also assumes that the student will not only write the letter shapes but will also be able to subject these symbols to the symmetry operations that perform the logical calculations. This point amounts to a fair warning, because too

often I have been told about the uselessness of X-stem, usually by someone who knows how to write the letter shapes but who has not yet learned how to operate with them --- like Gerbert, not even knowing that this is possible.

The fourth major challenge does not have a clear parallel in the use of Hindu-Arabic numerals, so for this one we cannot look to PME for guidance. On the logic side, standard notations for the Propositional Calculus have evolved so that the main concentration is now being given to only three of the binary connectives (and, or, if), in contrast to the 16 letter shapes in X-stem. Often reasons based on axiomatics are given to justify that this logic can be done with as few as only one connective, such as with "Nor" alone, or with "Nand" alone. I call this "Axiomatics in Overdrive." Note that as long as we continue with this mindset and remain walled in by this dead end, fully at the mercy of the misplaced use of the full power of axiomatics, there will be no PLE that is worthy of the deeply grounded, systemic advantages of the X-stem Logic Alphabet.

In summary, I have laid out four conditions that I count as minimum to establish a good beginning for PLE. First, shift to another notation, one that, if it is not X-stem, is at least *in the same class* as X-stem. Second, call on sensorimotor games and the hand-held symmetry models to construct the logic curriculum for K to 12. Third, make sure to follow through enough to get on the other side of the Gerbert Barrier. Fourth, explore the full range of X-stem. This is also saying, do not remain both trapped by and restricted to a limited notation, much like wearing a straitjacket, this one imposed by the full force of Axiomatics in Overdrive.